

# ON THE INFINITELY GENERATED LOCUS OF FROBENIUS ALGEBRAS OF RINGS OF PRIME CHARACTERISTIC

ALBERTO F. BOIX

Let  $R$  be a commutative Noetherian ring of prime characteristic  $p$ , and let  $M$  be an  $R$ -module. For each integer  $e \geq 0$ , we denote by  $\text{End}_{p^e}(M)$  the set of  $p^e$ -linear maps of  $M$ ; that is,  $\text{End}_{p^e}(M)$  is made up by abelian group endomorphisms  $M \xrightarrow{\phi} M$  such that

$$\phi(rm) = r^{p^e} \phi(m) \text{ for all } (r, m) \in R \times M.$$

In this way, one can cook up the so-called Frobenius algebra of  $M$

$$\mathcal{F}^M := \bigoplus_{e \geq 0} \text{End}_{p^e}(M),$$

where multiplication is given by composition of maps.

The following question is, to the best of our knowledge, quite open:

*Question 0.1.* Let  $R$  be a commutative Noetherian ring of prime characteristic  $p$ . Is it true that

$$\overline{W^R} := \{\mathfrak{p} \in \text{Spec}(R) : \mathcal{F}^{E_{\mathfrak{p}}} \text{ is finitely generated as a ring over its degree zero piece}\},$$

where  $E_{\mathfrak{p}}$  is the injective hull of the residue field of  $R_{\mathfrak{p}}$ , is an open set?

The goal of this talk is to study Question 0.1 in some detail; more precisely, let  $S = R/I$  a commutative Noetherian ring which is a quotient of a regular Noetherian ring  $R$  (that is, locally regular) of prime characteristic  $p$ . Under these assumptions, we show that  $\overline{W^S}$  is closed under generalization; moreover, when  $S$  is a Stanley–Reisner ring, we will show that  $\overline{W^S}$  is really an open set in the Zariski topology, and provide an algorithmic and explicit description of the defining ideal of its closed complement.

The content of this talk is based on joint work with Danny A. J. Gómez Ramírez and Santiago Zarzuela [BGRZ].

## REFERENCES

[BGRZ] A. F. Boix, D. A. J. Gómez-Ramírez, and S. Zarzuela. On the infinitely generated locus of Frobenius algebras of rings of prime characteristic. Available at <https://arxiv.org/pdf/2203.08511.pdf>. 1

IMUVA–MATHEMATICS RESEARCH INSTITUTE, UNIVERSIDAD DE VALLADOLID, PASEO DE BELEN, S/N, 47011, VALLADOLID, SPAIN.

*Email address:* `alberto.fernandez.boix@uva.es`

---

A F. Boix is partially supported by Spanish Ministerio de Economía y Competitividad grant PID2019-104844GB-I00.