EXPLORING EXTENSIONS OF DIRICHLET AND GREEN-TAO THEOREMS ON ARITHMETIC PROGRESSIONS OVER POLYNOMIAL AND FORMAL POWER SERIES RINGS

ALBERTO F. BOIX AND DANNY A. J. GÓMEZ-RAMÍREZ

Dirichlet's Theorem on primes in arithmetic progressions asserts that, if a and b are relatively prime positive integers, then there are infinitely many primes in the arithmetic progression $a(-)+b$ running over the positive integers. On the other hand, the so–called Green–Tao Theorem asserts that, given any $k \in \mathbb{N}$, there are coprime integers a and b such that all the numbers $a+b,\ a+2b, \ldots,\ a+kb$ are primes.

The goal of this talk is to explore the validity of Dirichlet's and Green–Tao' statements over polynomial and formal power series rings. More precisely, we will see that Dirichlet's Theorem also holds replacing the ring of integers by the polynomial ring $\mathbb{Z}[x_1, \ldots, x_n]$; in contrast, we will exhibit a counterexample for the validity of this statement over the formal power series ring $\mathbb{Z}[\![x]\!]$. On the other hand, we provide a Green–Tao type result that works over the polynomial ring $A[x_1, \ldots, x_n]$, where A can be either $\mathbb{Z}$ or a field of characteristic zero.

The content of this talk is based on [GRB26], where the reader can find all the details.

References

[GRB26] D. A. J. Gómez-Ramírez and A. F. Boix. On extensions of Dirichlet and Green–Tao theorems and Goldbach–Dirichlet representations over certain families of commutative rings with unity. *Math. Nachr.*, 229(1):117–128, 2026. 1

Department of Mathematics, Universitat Politècnica de Catalunya BarcelonaTech, Av. Eduard Maristany 16, 08019, Barcelona, Spain.

Email address: alberto.fernandez.boix@upc.edu

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Email address: daj.gomezramirez@gmail.com