TOTALLY REAL POINTS ON THE CURVE $x^5 + y^5 + z^5 = 0$

ALAIN KRAUS

Abstract. Let $\overline{\mathbb{Q}}$ be an algebraic closure of $\mathbb{Q}$ and $\mathbb{Q}^{tr}$ be the subfield of $\overline{\mathbb{Q}}$ obtained by taking the union of all totally real number fields. For any prime $p \geq 5$, let $F_p/\mathbb{Q}$ be the Fermat curve of equation $x^p + y^p + z^p = 0$. It is known that the set $F_p(\mathbb{Q}^{tr})$ of the points of F_p rational over $\mathbb{Q}^{tr}$ is infinite. How to explicit non-trivial points $(xyz \neq 0)$ in $F_p(\mathbb{Q}^{tr})$? It seems that the only points already known in $F_p(\mathbb{Q}^{tr})$ are those of $F_p(\mathbb{Q})$ and they are trivial. The main purpose of this talk is to present a result obtained recently on this question in case $p = 5$. I will also make some comments concerning the general case.