CONTRIBUTIONS TO THE STUDY OF CARTIER ALGEBRAS AND DIFFERENTIAL OPERATORS STNB 2015: COMMUNICATION PROPOSAL

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Introduced by M. Blickle and K. Schwede (see [BS13] and the references therein), the so-called *Cartier algebras* play an important role in the study of, on one hand, singularities in prime characteristic and, on the other hand, differential operators. Roughly speaking, a Cartier algebra is just a non-commutative ring on which one collects certain homogeneous functions of degree $1/p^e = p^{-e}$, where $e \geq 0$ is any non-negative integer.

From now on, let $\mathbb{K}$ be any finite field of prime characteristic p, let $S = \mathbb{K}[x_1, \dots, x_d]$ be the ring of polynomials in d variables with coefficients in the field $\mathbb{K}$, let $\mathcal{C}^S$ be the Cartier algebra attached to S, and let $\mathcal{D}_S$ be the ring of differential operators associated to S. The purpose of this talk is to introduce two algorithms related with Cartier algebras and differential operators. First of all, we describe a method which calculates all the ideals of S contained in $\langle x_1, \dots, x_d \rangle$ fixed with respect to a subalgebra of $\mathcal{C}^S$ generated by one homogeneous element. On the other hand, we provide a procedure which produces a differential operator $\delta \in \mathcal{D}_S$ such that $\delta(1/f) = 1/f^p$, i.e. a differential operator that acts as the Frobenius homomorphism on $1/f$; as a byproduct of this method, we describe a new characterization of ordinary and supersingular elliptic curves over $\mathbb{F}_p$. Moreover, we also explore the case of homogeneous quadrics (aka quadratic forms).

The content of this talk is based, on one hand, in a joint work with Mordechai Katzman (see [BK14] and [BK13]) and, on the other hand, in a joint work with Alessandro De Stefani and Davide Vanzo (see [BDSV]).

References

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