AN ALGORITHM FOR PRODUCING DIFFERENTIAL OPERATORS IN PRIME CHARACTERISTIC

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Let $R = k[x_1, \dots, x_d]$ be the polynomial ring in d variables over a field k, and let $f \in R$ be a non-zero polynomial. In any undergraduate Algebra course, it is shown that the localization R_f is not finitely generated as R-module. Now, let D be the ring of k-linear differential operators on R. It is known that the natural action of D on R extends to an action on R_f, hence R_f may be regarded as left D-module; in contrast with the infinite generation of R_f as R-module, it turns out that R_f is cyclic as D-module. More precisely, if k is a field of characteristic 0, then it is known that R_f is generated as D-module by $1/f^m$, where m is the greatest integer root in absolute value of the b-function attached to f, and there are examples where $m \geq 2$. On the other hand, if k is a field of prime characteristic p, it turns out that R_f is generated as D-module by $1/f$; the key ingredient for proving this striking fact was to show the existence of a differential operator $\delta \in D$ such that $\delta(1/f) = 1/f^p$.

The main purpose of this talk is to introduce a procedure which, given as input a polynomial f with coefficients in a finite field, produces as output a differential operator $\delta \in D$ such that $\delta(1/f) = 1/f^p$. We also study the level of such a differential operator (a concept that we define along the way) for some specific families of polynomials. In particular, we obtain a characterization of ordinary and supersingular elliptic curves over $\mathbb{Z}/p\mathbb{Z}$ in terms of the level of δ and, if time permits, we also exhibit some new results for hyperelliptic curves of arbitrary genus that show, in particular, that the level can still distinguish somehow between ordinary and supersingular (not superspecial) hyperelliptic curves.

The content of this talk is based, on the one hand, on joint work with Alessandro De Stefani and Davide Vanzo [BDSV15] and, on the other hand, on joint work with Iván Blanco–Chacón, Stiofáin Fordham, and Emrah Sercan Yilmaz [BCBFY18].

References

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