EULER'S FACTORIAL SERIES, HARDY INTEGRAL, AND CONTINUED FRACTIONS

TAPANI MATALA-AHO

ABSTRACT. Let p be a prime and let

$$E_p(t) = \sum_{k=0}^{\infty} k! t^k$$

denote the Euler's factorial series. We will present recent results on lower bounds for the p-adic absolute value of the expression $dE_p(p^a) - c$, where $a, c, d \in \mathbb{Z}$. The proofs are based on the fact that the same Padé polynomials which p-adically converge to $E_p(t)$, approach the Hardy integral

$$\mathcal{H}(t) = \int_0^{\infty} \frac{e^{-s}}{1-ts} ds$$

on the Archimedean side. Furthermore, we will discuss on an interconnection between $E(t)$ and $\mathcal{H}(t)$ via continued fractions.

The results are based on joint works with *Anne-Maria Ernvall-Hytönen, Louna Seppälä and Wadim Zudilin.*

TAPANI MATALA-AHO, DEPARTMENT OF MATHEMATICS, AALTO UNIVERSITY, FINLAND
E-mail address: tapani.matala-aho@aalto.fi