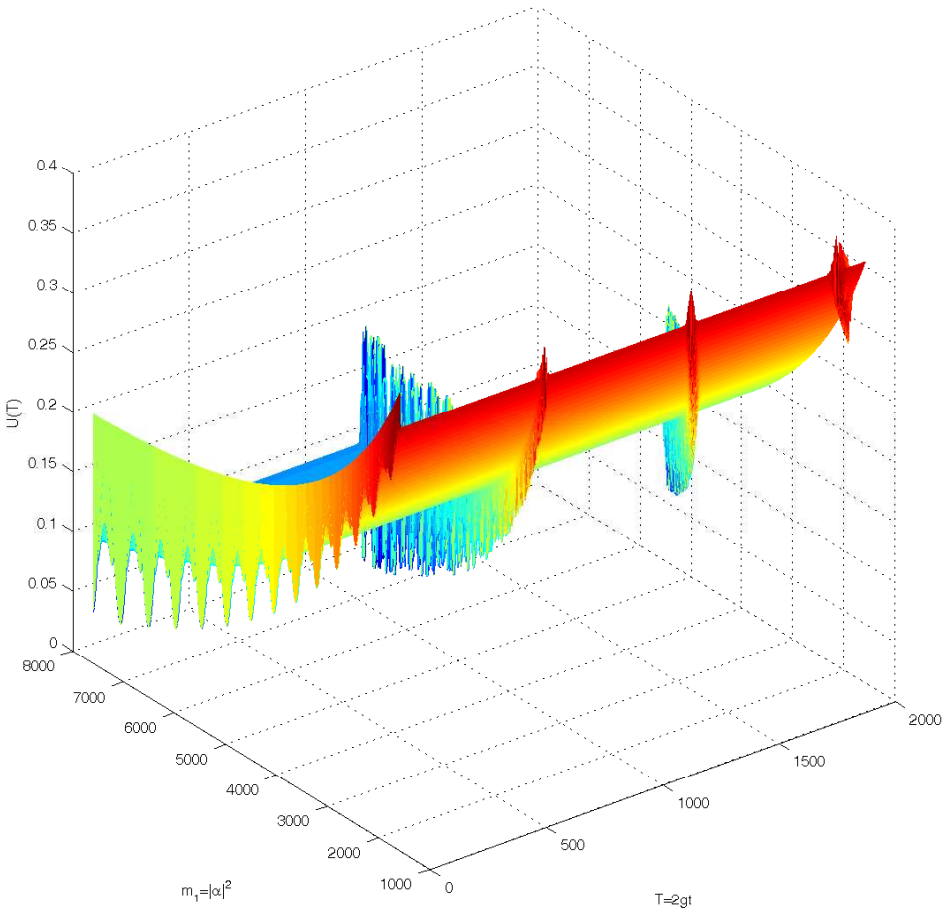
On approximation of special series of oscillating terms of quantum theory

Ekatherina A. Karatsuba

October 23, 2015

Dorodnicyn Computing Center of RAS
Vavilova 40, Moscow
119333 RUSSIA
email: karacuba@mi.ras.ru
FAX: +7-499-1356159



The Jaynes-Cummings model (JCM, see [1], [2], [3]) describes a single two-level atom interacting with a single mode of the quantized electromagnetic field in an ideal resonator. At first the JCM is considered as a pure theoretical model. However, the creation of one-atom maser and microlaser, and also one-mode resonator of high quality allowed to realize the JCM in practice. The data obtained in experiments confirmed some phenomena theoretically predicted in the JCM, for example, the collapses and revivals of the oscillations of atomic inversion. At present the JCM occupies a special place in quantum optics also for the reason that as experimentally confirmed it helps to examine and verify the conjectures about more complicated models.

Let the atom be in its excited state at $t = 0$, and the radiation field be in a coherent state with the Poissonian photon statistics. Then the atomic inversion $W = W(t)$ at a time $t > 0$ is defined by

$$W(t) = \sum_{n=0}^{+\infty} \frac{|\alpha|^{2n} e^{-|\alpha|^2}}{n!} \left(\frac{\Delta^2}{\Omega_n^2} + \frac{4g^2(n+1)}{\Omega_n^2} \cos(\Omega_n t) \right), \tag{1}$$

where $\Omega_n = \sqrt{\Delta^2 + 4g^2(n+1)}$ is the Rabi frequency; Δ is the detuning parameter that is, the difference between the frequency of atomic transition and the frequency of the field in the resonator; $\Delta \geq 0$; g is the interaction strength between the atom and the radiation; $g > 0$.

Here α is in general case a complex parameter, and also $|\alpha|^2$ is the initial average number of photons before the interaction of the field with the atom.

To approximate the sum (1) we construct a new method [4], [5], [6], [7] which is based on the techniques used in number theory. To get an asymptotic approximation of the inversion (1) which is as precise as possible, we investigate the series (1) and approximate it by various functions. Applying the functional equation of the Jacobi Theta function we get new asymptotic formulas for the atomic inversion. In this way we establish the relations between the parameters of the system guaranteeing the appropriateness of the obtained asymptotics.

New asymptotic formulas for the atomic inversion are found. These formulas give the possibility to predetermine the details of the behavior of the inversion on various time intervals and to establish the characteristics of the collapses and revivals of quantum oscillations depending on the parameters of the system.

References

[1] E. T. Jaynes and F. W. Cummings. Proc. IEEE, 51, 89–109 (1963).

[2] H. Paul. Annalen der Physik 466, 7-8, 411–412 (1963).

[3] M. O. Scully and M. S. Zubairy. Quantum Optics. Cambridge University Press (1997).

[4] E. A. Karatsuba, On an approach to the study of the Jaynes-Cummings sum in quantum optics, J. of Numerical Algorithms, 45:1-4, 127–137 (2007).

[5] A. A. Karatsuba, E. A. Karatsuba. Application of ATS in a quantum-optical model. Analysis and Mathematical Physics: Trends in Mathematics, Birkhäuser Verlag, Basel, 211–232 (2009).

[6] A. A. Karatsuba, E. A. Karatsuba. Resummation formula for collapse and revival in the Jaynes-Cummings model. J. Phys. A: Math. Theor., 42, 195304, 1–16 (2009).

[7] A. A. Karatsuba, E. A. Karatsuba. On Application of the Functional Equation of the Jacobi Theta Function to Approximation of Atomic Inversion in the Jaynes-Cummings Model. Pacific Journal of Applied Mathematics, 2:3, 41–63 (2010).