

SERRE'S CONSTANT OF ELLIPTIC CURVES OVER THE RATIONALS

HARRIS B. DANIELS AND ENRIQUE GONZÁLEZ-JIMÉNEZ

ABSTRACT. Let E be an elliptic curve without complex multiplication defined over the rationals. The purpose of this talk is to define a positive integer $A(E)$, that we call the *Serre's constant associated to E* , that gives necessary conditions to conclude that $\rho_{E,m}$, the mod m Galois representation associated to E , is non-surjective. In particular, if there exists a prime factor p of m satisfying $\text{val}_p(m) > \text{val}_p(A(E))$ then $\rho_{E,m}$ is non-surjective. We determine all the Serre's constants of elliptic curves without complex multiplication over the rationals that occur infinitely often. Moreover, we give all the possible combination of mod p Galois representations that occur for infinitely many non-isomorphic classes of non-CM elliptic curves over \mathbb{Q} , and the known cases that appear only finitely. We obtain similar results for the possible combination of maximal nonsurjective subgroups of $\text{GL}_2(\mathbb{Z}_p)$. Finally, we conjecture all the possibilities of these combinations and in particular all the possibilities of these Serre's constant.

DEPARTMENT OF MATHEMATICS AND STATISTICS, AMHERST COLLEGE, MA 01002, USA

E-mail address: `hdaniels@amherst.edu`

URL: `http://hdaniels.people.amherst.edu`

UNIVERSIDAD AUTÓNOMA DE MADRID, DEPARTAMENTO DE MATEMÁTICAS, MADRID, SPAIN

E-mail address: `enrique.gonzalez.jimenez@uam.es`

URL: `http://matematicas.uam.es/~enrique.gonzalez.jimenez`

Date: December 14, 2018.

2010 Mathematics Subject Classification. Primary: 11G05; Secondary: 11F80.

Key words and phrases. Elliptic curves, rationals, Galois representation.