

466 subgroups  $G \subseteq \mathrm{GL}_2(\mathbb{F}_5)$

$|\mathrm{GL}_2(\mathbb{F}_5)| = 480$

Divided in 48 conjugacy classes;  $G$  non-abelian with  $\det G$  surjective:

$\times$	GAP = label	LMFDB	5-cyc
	$[16, 6] = 8T7 = M_4(2)$	8 264	8
$\times 4$	$[20, 3] = 5T3 = F_5$	33 276	1 323
	$[24, 1] = 24T8 = C_3 \rtimes C_8$	0 (1 B)	even
	$[32, 11] = 8T17 = C_4 \wr C_2$	43 178	5
$\times 2$	$[40, 12] = 10T5 = C_2 \times F_5$	8 064	4
	$[48, 5] = 24T31 = C_8 \rtimes S_3$	0 (1 B)	0
	$[80, 30] = 20T20 = C_4 \times F_5$	1 640	3
	$[96, 67] = 24T138 = U_2(\mathbb{F}_3)$	20	0
	$[480, 218] = 24T1353 = \mathrm{GL}_2(\mathbb{F}_5)$	3 464	0

$$G = [48, 5] = 24T31$$

$$G = \langle a, b \mid a^2 = b^{24} = b^5 a b^{-1} \rangle = \left\langle a = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \right\rangle \subset \mathrm{GL}_2(\mathbb{F}_5)$$

$$L : x^{48} + 3972128 x^{32} + 49408 x^{16} + 1048576$$

$$K : x^4 + 4x^2 + 2$$

$$\begin{array}{ccccccc}
 & & & & \mathrm{Gal}(L/\mathbb{Q}) & & \\
 & & & & \downarrow & & \\
 & & & & \Downarrow & & \\
 & & & & \mathrm{Gal}(K/\mathbb{Q}) & & \\
 & & & & \downarrow \psi & & \\
 1 & \longrightarrow & \ker(\det) & \longrightarrow & G & \xrightarrow{\det} & \mathbb{F}_5^* \longrightarrow 1 \\
 & & & & \nwarrow e & & \uparrow \\
 & & & & & & 
 \end{array}$$

$$\rho: \text{Gal}(L/\mathbb{Q}) \longrightarrow G \subset \text{GL}_2(\mathbb{F}_5)$$

$$G = \left\langle a = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix} \right\rangle \subset \text{GL}_2(\mathbb{F}_5)$$

$$L: x^{48} + 3972128x^{32} + 49408x^{16} + 1048576$$

$$\Delta_L = 2^{192} \cdot 5^{32}$$

Higher ramification groups:

$$\text{At } 2: [16, 6]^{[4]} \supseteq C_8^{[3]} \supseteq C_4^{[4]} \supseteq C_2^{[8]} \supseteq 1: N = 2^{10}$$

$$\text{At } 5: C_3 \times C_8 \supseteq C_3 = \langle b^8 \rangle \supseteq 1 \quad : k = 9$$

$$\text{Modular tag for } L/\mathbb{Q}: f(q) = \sum_{n=1}^{\infty} a_n q^n \in S_9(\Gamma_1(2^{10}))$$